Vortex Breakdown in Swirling Conical Flows

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This paper describes some experiments in swirling flows in a mildly diverging cylindrical tube in which three types of vortex breakdown were observed: double-helix and spiral forms (followed by turbulent mixing), and axisymmetric form (often followed by a spiral breakdown, then by turbulent mixing). The type and location of the breakdowns were found to be dependent upon the Reynolds and circulation numbers of the flow. The observations reported and the evidence presented herein revealed that the axisymmetric breakdown is basically a finite transition between two sequent states of flow, from a uniform state of swirling flow (supercritical) to one (subcritical) featuring a series of standing waves of finite amplitude. The double-helix and spiral forms, which occur in a region well defined by Reynolds and circulation numbers, appear to be a consequence of the instability of the vortical viscous flow to spiral disturbances.

Introduction

VORTEX breakdown (an abrupt change in the structure of the core of a swirling flow) has been observed to occur over delta wings[1-10] at large incidences and in axisymmetric swirling flows in tubes.[11,12] In either case, its occurrence is marked by a rapid deceleration, deformation, and/or expansion of the core, flow reversal, and changes in the velocity and pressure distribution in the surrounding swirling flow. These changes do, in turn, cause, particularly for wings of small aspect ratio, significant changes in the slopes of lift, drag, and moment-coefficient curves. The practical significance of the phenomenon does not come solely from its unfavorable effect on delta wings. In fact, the breakdown may occur under a variety of other circumstances such as in suction tubes of pumps with inducers, draft tubes of turbines, cyclone separators, in trailing vortices behind a wing with a vortex dissipator, etc., and influences the performance of the system under consideration.

The breakdown seems to occur in three modes, the first in which the vortex core expands and spirals into a double helix, the second in which the core assumes a spiral form, and the third in which the core expands into an axially symmetric, nearly closed form. The first mode has been observed only and recently by Sarpkaya[12] in a diverging tube. Of the remaining two forms, the spiral form is more readily observed over wings and the axisymmetric form in tubes, even though both forms may occur, individually or simultaneously, both in tubes and over wings.

The practical as well as the theoretical significance of the phenomenon led to the advancement of several theoretical explanations. Most of these theories and the experimental work carried out prior to 1966 have been reviewed by Hall.[13] The present paper, which is the sequel of the author's previous publication,[12] presents a brief review of some of the recent work, then describes the additional observations made, and finally discusses the results in the context of the existing theories.

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A Brief Review of the Previous Theoretical and Experimental Works

Squire,[14] whose inviscid flow model formed the basis of many later works, connected the axisymmetric vortex breakdown with the appearance of infinitesimal standing waves in the flow. For this purpose, Squire reduced the equations of motion for an axisymmetric flow to

$$D^2\Psi = -K(dK/d\Psi) + (r^2/\rho)(dP_0/d\Psi) \qquad (1)$$

where Ψ is the Stokes stream function and $K \equiv vr$. P_0 is the total pressure and, like the circulation, remains constant along any stream tube. Thus, the evolution of an axisymmetric swirling flow may be calculated through the use of the appropriate boundary conditions provided that $K(\Psi)$ and $P_0(\Psi)$ are known in some region of the flow where $\Psi = \Psi_0$. This simple mathematical principle cannot, unfortunately, be applied to any physical situation without introducing some assumptions regarding the flow conditions at one axial station. In fact, it is partly this difficulty that led to the multiplicity of existing solutions. As will be noted later, the experimental determination of the velocity components and pressure at any axial distance z proved to be very difficult. Because the breakdown in axisymmetric flows takes place well within the region of flow development (entrance region) and the development of the flow strongly depends on the initial upstream conditions (centerbody, swirl vanes, transition piece, etc.), a sufficiently precise identification of the occurrence and location of the vortex breakdown through Eq. (1) does not yet seem possible.

Squire, assuming $\Psi_0 = \frac{1}{2}W_0r^2$, i.e., the axial velocity $W_0 =$ const, and $\Psi = \Psi_0 + \psi$, where now ψ is a small perturbation in flux, reduced Eq. (1) to

$$D^2\psi + (\psi/2\psi_0)(dK^2/d\psi_0) = 0 \qquad (2)$$

The use of three different, physically plausible swirl velocity distributions then yielded the following maximum swirl angles for which long standing waves can be sustained:

$$v = \omega r \quad r \leq r_c \qquad v = \omega r_c^2/r \quad r \geq r_c \qquad \phi_{max} = 50.2°$$

$$v = (\omega r_c^2/r)[1 - \exp(-r^2/r_c^2)] \qquad \phi_{max} = 45°$$

$$v = \frac{\omega r_c^2}{r}\left[\int_0^{r^2/r_c^2} y^2 \operatorname{sech}^2 y \, dy\right]^{1/2} \qquad \phi_{max} = 49.3°$$

The maximum swirl angle predicted by Squire has been the starting point in the design of almost every experimental apparatus for the investigation of the vortex breakdown phenomenon.

Benjamin[15] pointed out that the waves discussed by Squire cannot spread upstream from a disturbance, since the group velocity of these waves is directed downstream. Instead, Benjamin proposed that vortex breakdown is a finite transition between two dynamically sequent states of axisymmetric flow, analogous to the hydraulic jump in open-channel flow. This concept may be illustrated by applying it to the first swirl-flow condition considered by Squire, namely, that $v = \omega r, r \leq r_c$; and $v = \omega r_c^2/r, r \geq r_c$; and $\Psi_0 = \frac{1}{2}W_0 r^2$.

For $R > r \geq r_c$ (annular region), one has $K = \omega r_c^2$ and $P_0 = P + \frac{1}{2}\rho W_0^2 + \rho\omega^2 r_c^2$. Equation (1) then reduces to $D^2\Psi = 0$. Thus the equation of continuity between the two sequent states of uniform flow becomes

$$(R^2 - r_c^2)W_0 = (R_s^2 - r_{sc}^2)W_{s0}$$

For $r \leq r_c$ (core region), one has $K = 2\omega\Psi/W_0$ and $P_0 = P + \frac{1}{2}\rho W_0^2 + 2\rho\omega^2\Psi/W_0$. Together with Eq. (1), one obtains

$$D^2\psi = (4\omega^2/W_0^2)(\tfrac{1}{2}W_0 r^2 - \psi) \qquad (3)$$

Equation (3) is now solved with the boundary conditions $r = 0$, $\psi = 0$, and $r = r_{sc}$, $\Psi = \frac{1}{2}W_0 r_c^2$, which, after considerable simplification, yields

$$\frac{R^2 - r_c^2}{R_s^2 - r_{sc}^2} = 1 + \left(\frac{r_c^2}{r_{sc}^2} - 1\right)\frac{\omega r_{sc}}{W_0}\frac{J_0(2\omega r_{sc}/W_0)}{J_1(2\omega r_{sc}/W_0)} \qquad (4)$$

For $R = R_s$ (uniform tube) and as $r_c \rightarrow r_{sc}$, the expression for the maximum swirl angle from Eq. (4) reduces to

$$\frac{J_0(2\tan\phi_{\max})}{J_1(2\tan\phi_{\max})} = \frac{1}{[(R/r_c)^2 - 1]\tan\phi_{\max}} \qquad (5)$$

where $\tan\phi_{\max} = \omega r_c/W_0$. It is evident from the development of Eq. (5) that the critical condition depends on the particular primary flow chosen and that Benjamin's analysis yields no simple criterion for the conditions leading to breakdown.

Benjamin's analysis is not, however, restricted to any particular type of cylindrical support flow. The general analysis, as well as the foregoing example, shows that the momentum flux of the sequent flow does not remain equal to that of the primary flow. An additional mechanism is needed to cause a reduction in momentum flux. Benjamin hypothesized that the difference in momentum flux is accounted for by standing waves superimposed on the sequent flow, provided that the difference is rather small. The theory does not apply to cases where the momentum difference is large, since the waves are now replaced by turbulence and the total pressure and circulation are no longer conserved along the stream tubes. Even though steady breakdowns involving only small perturbations had not yet been experimentally observed, Benjamin's theory[15–17] has certain remarkable features that uniquely explain some of the observations to be discussed later.

The vortex breakdown has been viewed by Ludwieg[18,19] as a consequence of instability to spiral disturbances of an inviscid flow spiralling in a narrow annulus. According to Ludwieg, the flow will be unstable if

$$(\tfrac{5}{3} - f)g^2 - (1 - f)(1 - f^2) > 0$$

where

$$f(r) = \frac{r}{v}\frac{\partial v}{\partial r} \qquad g(r) = \frac{r}{v}\frac{\partial u}{\partial r}$$

Firstly, this theory is, as cited previously, restricted to inviscid flows in a cylindrical annulus with a narrow gap. Secondly, it cannot explain the axisymmetric type of breakdown, since Jones[20] has shown that the original flow is of a kind that is highly stable to axisymmetric disturbances. Thirdly, as Hall[21] has pointed out, Ludwieg's test criterion predicts instability for a wide range of quasi-cylindrical flows, including those that do not exhibit vortex breakdown. In spite of these objections, it appears, on the basis of the observations reported herein, that the stability argument has some basis for flows with small Reynolds and large circulation numbers.

Hall[13,21] explored the analogy between breakdown and boundary-layer separation by applying a boundary-layer type of approximation to the Navier-Stokes equations for quasi-cylindrical viscous flows. He found, by a step-by-axial-step integration scheme, that the solution begins to develop, at some position along the axis, strong axial gradients for flows with sufficiently strong circulation and/or adverse pressure gradient. This result is interpreted by Hall as a prelude to stagnation on the axis and eventual vortex breakdown. Hall's analysis cannot predict what happens beyond the stagnation point or why the flow behind an axisymmetric breakdown returns once again to a state almost identical to that prior to breakdown. His analysis has, however, the advantage over that of Benjamin that it predicts an axial station at which breakdown is likely to occur. In the conjugate flow theory, the flow prior to transition is assumed to be independent of axial variations. Thus the approach of flow to the critical state may be evaluated only through the use of experimentally determined velocity profiles.

Hall applied, to the velocity profiles he calculated, the breakdown criteria of Ludwieg and Benjamin. He concluded, within the limitations and interpretations of various criteria, that the finite transition theory, stability theory, and the criterion of imminent stagnation on the axis can serve equally well to identify the failure of the quasi-cylindrical approximation.

Bossel[22] made an attempt to analyze the vortex breakdown flowfield by reducing the equations of motion to simpler sets in four different regions of the flowfield. He claimed to have obtained solutions that agree in all important aspects with observations but did not comment on the differences between Harvey's measurements and his calculations. He concluded that the vortex breakdown is a necessary feature of supercritical viscous-vortex flow having high swirl close to the critical state and some flow retardation at and near the axis and that neither the finite-transition nor the hydrodynamic-stability theory appears justified. A critical examination of Bossel's results shows that his predicted swirl velocities are approximately half those measured by Harvey[11] and Sarpkaya[12] at the corresponding points; that the size of the bubble predicted by him is quite variable (with the values assigned to the disposable parameters), even though the shape of his bubble remains practically constant under a wide range of conditions; that the observed bubbles are not closed, contrary to his prediction; and that Bossel's conclusions regarding Benjamin's finite transition theory and Ludwieg's stability theory do not appear to be justified in view of the observations presented herein.

Leibovich[23] examined the behavior of weakly nonlinear waves in rotating fluids through the use of the Korteweg-de Vries equation and the several cases of critical stationary flows with reference to Benjamin's finite transition theory. He has shown that infinitesimal, critical waves will not exist in a Poiseuille type of support flow with solid body rotation. Even though such flows are known to be stable to axisymmetric disturbances and unstable to nonaxisymmetric disturbances (Pedley[24]), it is not known whether the *developing Poiseuille* flow with solid body rotation is stable to any one of these two types of disturbances. For flows with large Reynolds numbers, however, the flow in the entrance region (with the exception of a thin boundary layer) may be regarded as essentially uniform. For such flows, Leibovich was able to

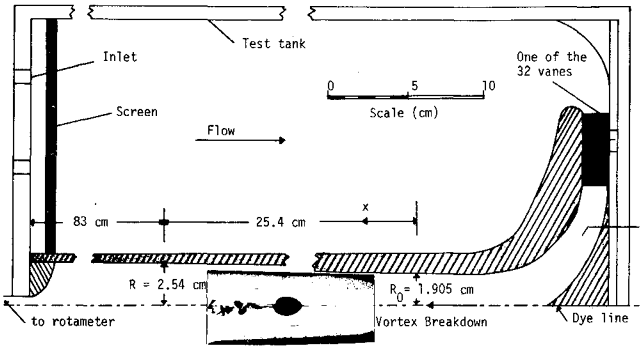

Fig. 1 Top half of the experimental equipment.

predict the shape of the breakdown bubble, even though the relative diameter of his bubble is approximately half that observed experimentally.

The experimental investigations of the vortex-breakdown phenomenon have been mostly qualitative. Among the notable are those by Werle,[1] Elle,[2] Harvey,[11] Lambourne and Bryer,[6] Hummel and Srinivasan,[10] and Sarpkaya.[12] Harvey observed an axisymmetric breakdown consisting of a bubble that showed no signs of a reversed core and a swirling flow downstream which appeared rather to have returned to a form similar to that upstream of the breakdown. Harvey's experiments did not provide any detailed information regarding the structure of flow in the breakdown bubble. Sarpkaya's[12] experiments were directed to the study of the characteristics of the breakdown bubble and to the observation of the motion of travelling breakdowns. The present work, confined to stationary breakdowns, was designed for the express purpose of understanding the role of instability in the initiation and subsequent development of all types of breakdowns.

Apparatus

A schematic diagram of the experimental apparatus is shown in Fig. 1. Swirl was imparted to the fluid by 32 streamlined foils placed symmetrically in a circular array around the inlet piece. The simultaneous rotation of the vanes set the desired flux of angular momentum entering the test tube.

The flow rate was controlled independently of the swirl vanes with a valve and flow meter placed at the downstream end of the tube. Discharge was varied from 0.05 to 2 ft^3/sec, and the vane angle, between the swirl vanes and a radial line, was varied continuously from 0° to 60°. For additional details, the reader is referred to Ref. 12.

Observations and Measurements

Spiral and Double-Helix Breakdowns

A known flow rate through the test tube was established, and dye was introduced on the tube centerline through the centerbody. The dye filament maintained a perfectly straight and laminar form throughout the length of the tube for a 0° vane-angle setting. As the swirl vanes were adjusted to produce a small flux of angular momentum, spiralling waves developed toward the end of the diverging tube, and

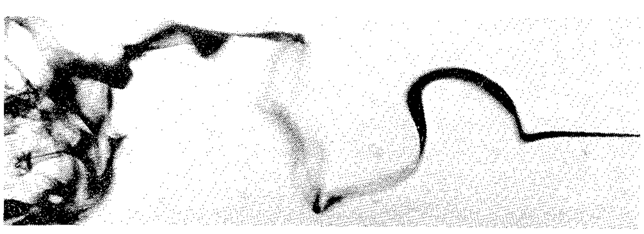

Fig. 2 Spiral breakdown.

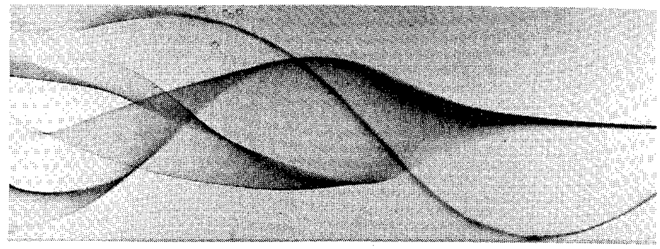

Fig. 3 Double-helix breakdown (Re = 1700, Ω = 2.35), D_0 = 3.81 cm).

the filament became sheared into a tape that broke into turbulence after several revolutions in the form of a helix. As the vane angle was increased further, the filament decelerated rapidly near the midlength of the tube and deformed, following an abrupt kink, into a spiral configuration. The spiral persisted a few turns and then broke into large-scale turbulence, as seen in Fig. 2. The sense of rotation of the spiral was identical to that of the fluid surrounding the original filament.

Subsequent increases in swirl resulted in two distinctly different forms of breakdown. The occurrence of either one of these forms depended to a large extent on the Reynolds number† and the rate of increase of swirl. The first and more commonly observed form was the expansion and folding of the filament, before the completion of the first turn, all the way back to the kink. Further increase of swirl produced an almost axisymmetric bubble.

The second and less commonly observed form occurred only for Reynolds numbers less than about 2000 and circulation numbers† larger than 2.3. In this form, the original filament gradually decelerated and expanded into a slightly curved triangular sheet, as seen in Figs. 3 and 4. Each half of the continuous sheet wrapped around the other (rotating in the same sense) into the form of a double helix. This form gradually expanded, filled the entire test tube, and subsequently broke into a very mild turbulence.

The double-spiral mode, never observed in the previously reported studies (except for Sarpkaya[12]), appeared to occur and sustain itself with very little energy loss. Once it came into being, its appearance and location were quite steady, and the intermittent injection of the dye did not disturb it. It is significant to note that the filament rapidly decelerated but did not stagnate. The double-helix form was, however, quite sensitive to upstream as well as downstream disturbances, and the slightest fluctuation in the flow or the vibrations emitted by the external system forced it gradually to deform and take indescribably complex forms or to rotate

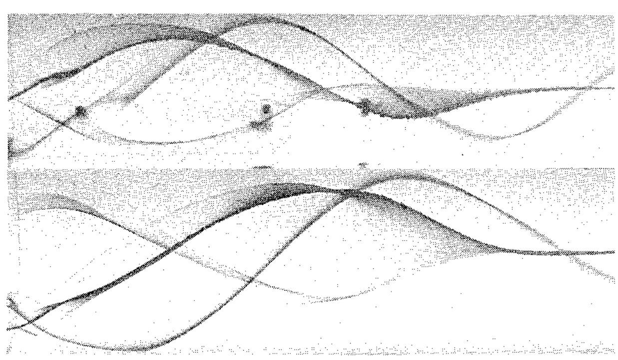

Fig. 4 The top and front views of another double-helix form and the off-center filament.

† Reynolds number $Re = U_0D_0/\nu$, circulation number $\Omega = \Gamma/U_0D_0$. U_0 = mean velocity, where $D = D_0$ (diameter of the tube at the start of divergence), and Γ circulation imparted to the flow) $= 2\pi R_i V_i \sin\beta$. R_i = radial distance to the tip of a vane, and V_i = the uniform velocity of flow between any two vanes for a given flow rate and vane-setting angle β.

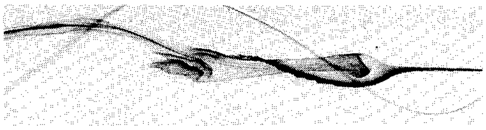

Fig. 5 Inception of a bubble via double-helix form.

relative to the tube and gradually break down in the form of many secondary spirals.

The previous study[12] did not dwell further on this type of breakdown and concentrated rather on the axisymmetric type of breakdown. The additional observations were made for the purpose of understanding the nature of this type of breakdown and answering certain questions that have arisen as a result of its discovery. It was conjectured that it might be an incipient state of breakdown formation where stream surfaces abruptly expand near the axis. It was also conjectured that the double helix might represent not the central dye filament but, in fact, one that is slightly displaced to the side, and that the expansion into a triangular sheet occurs as this displaced filament passes around the edge of the unobserved but, nevertheless, possibly present breakdown bubble. These possibilities were carefully investigated by injecting dye very near the core, by displacing the centerbody in all possible directions, and by introducing dye into the center region of the expanding sheet. The results have conclusively shown that there is neither a bubble nor a stagnation point. Furthermore, it was observed that the sheet expands, gradually fills the entire tube, and breaks into mild turbulence without ever returning to its original form upstream of the breakdown provided that the Reynolds and circulation numbers remain within the limits cited previously.

This writer was initially tempted to regard the double-helix form as a vortex breakdown as it is commonly understood. Because this type of transition is highly sensitive to small disturbances, because it gradually expands and breaks up into turbulence (until there is no longer any detectable swirl), because it does not produce a bubble, and because it occurs at relatively low Reynolds and high circulation numbers, the writer is inclined to conclude that it is a consequence of the instability of the prevailing flow to spiral disturbances. The double-helix transition then represents a new phenomenon, and it is akin to the growth of familiar disturbances in nonswirling flows. Thus, it is suggested that one possible form of the instability for such flows resembles, when its amplitude becomes sufficiently large, the double-helix form observed in this investigation.

There appears to be some evidence for the preceding conjecture. Linear viscous analyses for rotating Poiseuille flow have demonstrated that profound rotation-induced changes take place in the flow when the rate of rapid rotation is suitably adjusted. Talbot,[25] in studying the decay of a rotationally symmetric steady swirl superimposed on Poiseuille flow in a round pipe, observed that there were two markedly different types of instability. For Reynolds numbers less than about 1800, the instability of the swirl is characterized by seemingly nonperiodic sinuous motion of the dye filament. For Reynolds numbers in excess of about 2500, Talbot observed that the instability of the swirl appeared to begin as a series of eddies of definite spatial frequency, somewhat like the Taylor-type instability found under certain conditions in the annulus between two concentric rotating cylinders. Talbot determined a critical curve for swirl instability in the range of Reynolds numbers from 200 to 4000 and circulation numbers from approximately 1 to 60. He did not, however, determine the location of the instabilities, nor did he elaborate further on their form. It now appears that the range of Reynolds and circulation numbers encountered in his work could not have permitted him to observe an axisymmetric breakdown.

Clearly, although the theoretical problems involved in a complete description of the observed modes of transition are formidable, the rather specialized stability analyses on related flows by Ludwieg,[18] Pedley,[24] Joseph and Munson,[26] and Talbot[25] appear more relevant to the observations reported herein than the analyses proposed by Benjamin and others.[21–23]

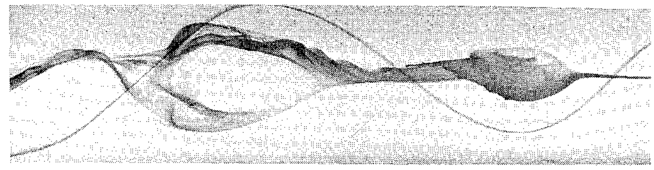

Fig. 6 Axisymmetric, standing, breakdown bubbles.

Axisymmetric Breakdown

This type of breakdown evolved either from a double helix, or from a spiral, or directly from an axisymmetric swelling of the vortex core. The mode of evolution depended, to varying degrees of intensity, on the particular combination of the Reynolds and circulation numbers.

For Reynolds numbers less than about 2000 and circulation numbers of about 2.5, the axisymmetric breakdown often evolved from a double-helix form. One part of the double-spiralling sheet wrapped around rather tightly and moved upstream and formed an almost closed cylindrical surface, as seen in Fig. 5. The slight increase of the momentum flux then resulted in a vortex-breakdown bubble, as seen in Fig. 6. The flow downstream of the bubble returned, with a thicker core, to its original state and then formed another egg-shaped large bubble with no tendency to break up into turbulence. In fact, this alternating form of the vortex core continued some distance downstream until the dye completely diffused.

Further increase of flow and/or swirl produced a nearly axisymmetric bubble, as in Fig. 7. After a distance of approximately one bubble length, the new core downstream of the bubble often deflected, following an abrupt kink, into a loose spiral configuration. Under rare circumstances, however, the new core formed another stationary but relatively smaller bubble, as in Fig. 7.

For Reynolds numbers larger than approximately 2000 and in the region of "vortex-breakdown hysteresis" (to be discussed later in connection with Fig. 14), the axisymmetric breakdown evolved from a spiral form. A small increase in flow and/or swirl, following the establishment of a spiral breakdown, distorted the filament in such a manner that, after the first turn, it developed a tendency to curl back toward the kink. Subsequently, the spiral expanded and folded all the way back to the kink. Further increase of flow and/or swirl produced an axisymmetric bubble similar to those shown in Figs. 7 and 8. The latter photograph, which was taken shortly after the center-dye injection had been stopped, is clear evidence of the persistence of the dyed water in the bubble as well as in the flow pattern downstream of the bubble.

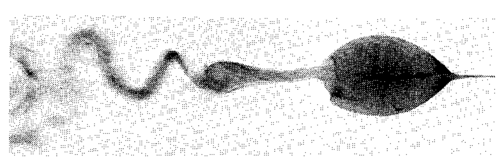

Fig. 7 Axisymmetric breakdown followed by a smaller bubble and spiralling motion.

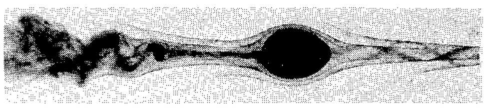

Fig. 8 Axisymmetric breakdown and surrounding stream surfaces.

Obviously, this could not have been possible had the bubble been closed and had the fluid in the spiralling core downstream of the bubble not been mixing with some fluid issuing from the bubble. It is now apparent that, for the dye in the bubble to fade away gradually rather than disappear abruptly, the bubble must continuously exchange fluid with its environment while operating on the vorticity of this fluid so as to preserve the local flow pattern. In fact, the motion pictures have shown that a toroidal vortex ring (captured at the downstream half of the bubble), whose axis gyrated at a regular frequency about the axis of the bubble, was primarily responsible for this simultaneous filling and emptying process. The fluid in the bubble replenished from the side of the bubble nearer the downstream portion of the ring and emptied from the side farther from the upstream portion of the ring. It is thus clear from the foregoing that the steady breakdown bubbles do not have a rear stagnation point and are characterized by an axial-flow reversal.

For sufficiently high Reynolds and circulation numbers, characterized only by an appropriate combination of the swirling-flow conditions (the left side of the hysteresis region), the axisymmetric breakdown bubble came into existence as a result of a perfectly axisymmetric swelling of the vortex-core filament. The number of the bubbles obtained at a given time was dependent on the rate of increase of circulation. When the flow rate and vane angle were properly set so as to arrive at a flow condition in the region defined previously, the dye filament expanded into an egg-shaped bubble similar to that shown in Fig. 8. Obviously, a steady bubble had been in existence there for quite some time, and the dye served no other purpose than to reveal its existence. The bubble did not change its location until either the flow or the swirl was changed.

When the initial flow condition was set (e.g., $Re \simeq 6500$, $\Omega \simeq 1.2$) so as to obtain only a spiral breakdown far downstream and then the swirl was rapidly increased (by increasing the vane setting by a few degrees), rarely one, often two, and at times as many as three axisymmetric bubbles came, bubble by bubble, into existence. The entire process was often completed in five or six frames of a motion picture taken at 64 frames/sec. No special attempt was made to determine the time interval between the formation of two successive waves (nor the group velocity) other than to note that the bubbles developed in succession of one another, the first being the farthest upstream, etc.

Figures 9–11 show the bubbles obtained in the manner just described. Figure 9 is too complex to describe in words. As noted earlier, the rapid increase of swirl seldom resulted in such a single bubble. The interpretation of the flow structure at its wake is left to the imagination of the reader.

Shortly after the formation of the bubbles shown in Figs. 10 and 11, the spiral form at the downstream end of the bubble train moved upstream into the last bubble and destroyed its symmetry and converted it into a spiral form. This process continued until the second bubble downstream

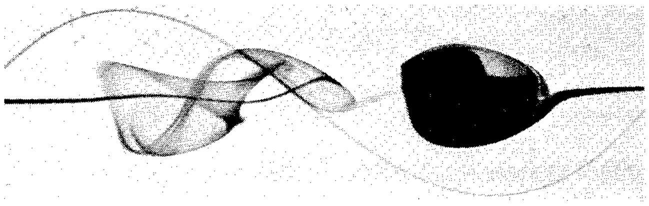

Fig. 9 The formation of a single bubble.

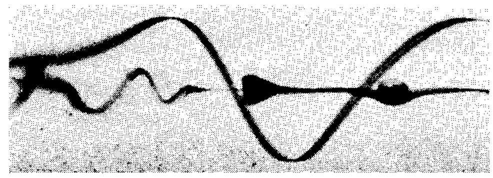

Fig. 10 The formation of two bubbles and the increase of swirl angle.

of the first was converted to a spiral form. The distance between the first bubble and the spiral remained essentially constant, i.e., the spiral did not move further upstream into the first bubble. In the meantime, the first bubble, which was closed and perfectly symmetrical during its formative stages, began to draw in fluid from its downstream end and grew in size so as finally to arrive at a form similar to that shown in Fig. 8. It is important to note that this bubble, during its growth period, moved upstream together with its spiralling tail and finally settled at a point where it would have occurred for the new swirl, had the latter been constant. To see that such an upstream propagation of a growing wave is perfectly reasonable in supercritical flows, one need only remember that the celerity of a wave increases w.th increasing amplitude, so that high waves in an open channel may travel upstream even though the Froude number exceeds the critical.

Three facts are by now apparent. Firstly, the axisymmetric form of breakdown is finally comprised of a large egg-shaped bubble, a thicker vortex core, and a spiralling tail regardless of its form of evolution. Secondly, the formation of a wave train, wave by wave, is in perfect accord with Benjamin's finite transition theory. No other theory, such as stability, impending stagnation, etc., can account for it. Finally, it is meaningless to talk of a subcritical flow between two bubbles just as it is meaningless to talk of a subcritical flow between two waves in an open channel flow for which the Froude number of the upstream supercritical flow[27] is close to unity. In other words, the subcriticality of the observed wavy flow exists only as a state and not as a measurable entity. Consequently, the condition predicted by Eq. (5) is only an idealization of the prevailing flow, since hypothetical, sequent, cylindrical flow is never realized.

Measurements

Figure 12 is a plot of a representative, stationary, axisymmetric breakdown bubble. Both axes were normalized with the local radius of the tube measured at the front stagnation point of the bubble. Various predictions and measurements of the bubble radius r_b are given in Table 1 in terms of the radial distance r_c (core radius) at which $\phi = \phi_{max}$.

Considering the fact that the bubble size is very sensitive to ϕ_{max} in Bossel's analysis and to a free parameter ϵ in Leibovich's work, any agreement or disagreement between the theoretical predictions and experimental observations should be interpreted with extreme caution. As to the differences in the size and shape of the bubbles observed in the present study and in Harvey's work, it appears that the divergence of the tube increases the diameter of the bubble. The effect of the Reynolds number/circulation number combination on

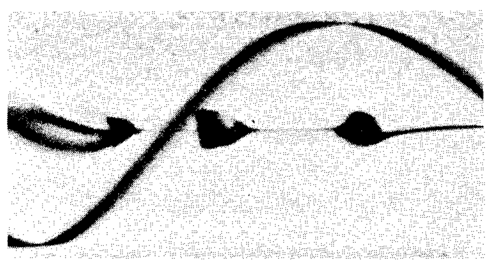

Fig. 11 The formation of three bubbles in rapid succession.

Table 1 Bubble radius predictions and measurements

Source	r_b/r_c	ϕ_{max}, deg
Harvey (experimental) (uniform tube)	$\simeq 0.63$	50.5
Present study (diverging tube)	$\simeq 0.90$	51
Bossel (theoretical)	$\simeq 0.23$	62.5
Leibovich (theoretical) (with Harvey's swirl data)	$\simeq 0.60$	46

the bubble size and shape has not yet been investigated. Preliminary observations showed that the bubble size decreases somewhat with increasing Reynolds numbers.

Figure 13 is a plot of the mean breakdown location. Evidently, the type and location of the stationary breakdown are functions of Reynolds and circulation numbers in the range of Reynolds numbers investigated and for the particular diverging tube used (D_0 = 3.81 cm); for smaller swirls, the axisymmetric breakdown occurs at higher Reynolds numbers; and finally, there is a region along every constant circulation line where there are two semistable breakdown conditions, i.e., there is a region of "vortex-breakdown hysteresis." As the flow rate was slowly increased while maintaining the vane angle setting fixed, the spiral breakdown moved upstream. Conversely, when the flow rate was slowly decreased in this region, the symmetric breakdown moved downstream. In the region of hysteresis, both forms were, as discussed previously, highly unstable and transformed into each other, depending on the instantaneous flow conditions. A similar hysteresis effect was observed by Lowson[8] over delta wings with increasing or decreasing incidence in the range of 35° to 45° angle of incidence.

Discussion of Results

It is apparent from the foregoing that, for sufficiently high Reynolds and circulation numbers, the axisymmetric vortex breakdown evolves from a symmetric swelling of the vortex core. Under these conditions, a wave train is established wave by wave. In time, the motion downstream of the first wave becomes unsteady and irregular. The formation of the vortex-breakdown bubble in this region is not preceded by the amplification of travelling-wave disturbances.

There is a region fairly well defined by certain values of Reynolds and circulation numbers in which only spiralling type of breakdown occurs. Immediately below that region and, in particular, for Reynolds numbers between approximately 1000 and 2000, there is a third region where the swirling flow is highly unstable to spiral disturbances. In this region, both the first and second modes of spiralling transition can be observed. Finally, there appears to be a fourth region in which the flow is stable regardless of the magnitude of circulation. These four regions are shown in Fig. 14, together with the critical curve for swirl instability obtained by Talbot.[25] Only a small portion of Talbot's curve could be included in this figure, since his circulation numbers varied from 1 to approximately 60. According to Talbot, the flow

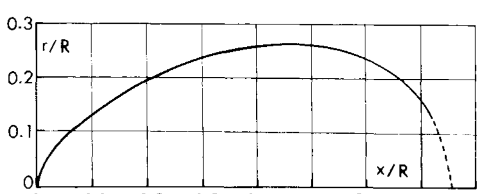

Fig. 12 The contour of a representative breakdown bubble.

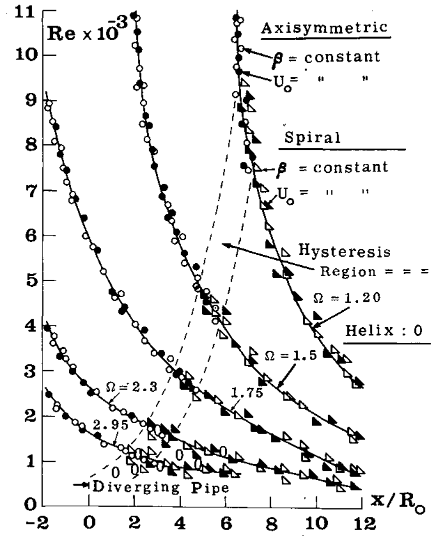

Fig. 13 Vortex-breakdown position as a function of Reynolds and circulation numbers.

in the region below this curve is stable. Even though there are not enought data to delineate sufficiently all of the four regions, it appears that the region in which the double-helix and/or spiral forms are observed in the present study coincides fairly well with the stability curve obtained by Talbot.

As noted in the text, the spiral breakdown may transform to axisymmetric form and vice versa in the hysteresis (or transition) region. To the right of this region, however, the waves form wave by wave. Even though it is difficult to define unambiguously the corresponding four regions cited previously in terms of four possible states of flow between supercritical, subcritical, and laminar and turbulent, it appears that the finite transition concept proposed by Benjamin is wholly in accord with the observations made in the *axisymmetric breakdown* region.‡ It also appears that the temporally periodic motion of the flow downstream of the first or subsequent bubbles is a consequence of the amplification of the disturbances generated in the subcritical flow (possibly by the

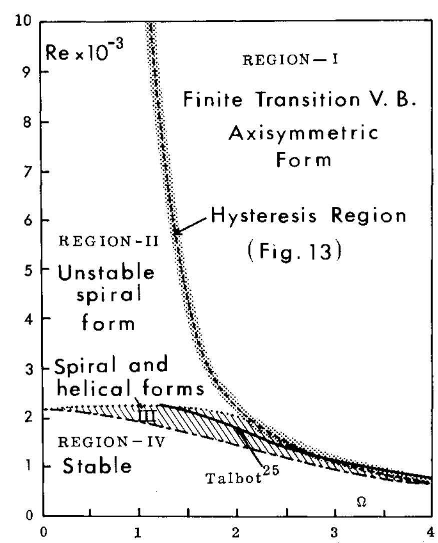

Fig. 14 The four states of swirling flow.

‡ In terms of the hydraulic-jump analogy, the finite transition with negligible energy loss appears to correspond to the wavy transitions in supercritical turbulent flows with Froude numbers less than about 1.7.[27] With the admission of a little more energy loss, the jumps occurring in the range of Froude numbers from 1.7 to 2.5 may also correspond to weak vortex breakdowns.

pressure instabilities in the wake and the highly unstable simultaneous filling and emptying process of the bubble) and not a result of the amplification of disturbances in the supercritical flow. Had the latter been true, it would not have been possible to obtain a series of bubbles as seen in Figs. 10 and 11, and the disturbances would have grown beyond bounds.

In the region of spiral breakdown seen in Fig. 14, it is rather difficult to determine whether the instability or the finite transition mechanism triggers the breakdown. It is possible, as conjectured by Benjamin,[17] that such an asymmetric, time-dependent spiral motion may arise from the instability of the steady wave trains (never observed in this region even when Ω was increased rapidly to a new steady value *below* the hysteresis region) and that the finite-transition theory may continue to describe the basic state upon which the actual flow subsists. In the experiments reported herein, the spiral breakdowns appeared to be quite sensitive to external disturbances, and their point of occurrence was fairly sensitive to small changes in flow and/or circulation. One could, for example, move the spiral upstream or downstream by slightly raising or lowering the tip of the centerbody. This latter action has invariably resulted in a kinked vortex core and observable small spirals near the point of dye injection, i.e., at the tip of the centerbody. In the region of axisymmetric breakdown, however, such a motion of the centerbody caused no changes either in the shape or the location of the axisymmetric bubble. It is, therefore, quite possible that the flow in the spiral breakdown region is inherently unstable and that the spiral does not necessarily arise from the instability of the finite-transition waves. Even though it is important to determine whether the growth of the disturbances give rise to spiral core (and to its eventual destruction) or the finite-transition waves cause the temporally periodic motion, it was not, in the present investigation, possible to resolve clearly this crucial issue. Nevertheless, the evidence presented, such as the growth of spirals beyond bounds, the strong effect of upstream disturbances, etc., leads us to believe that the instability of the swirling flow in the region II to spiral disturbances plays a dominant role in the breakdown of the vortex core.

As noted earlier, the third and fourth regions are even more difficult to define because of the complexity of making accurate measurements at sufficiently low Reynolds numbers in a test tube of rather limited length. Be that as it may, the double-helix form does not appear to be a vortex breakdown as it is commonly understood. It seems to be a consequence of the instability of the swirling flow (in Ludwieg's sense) to spiral disturbances. This type of instability appears to correspond to the instability of a supercritical laminar flow in a near-critical state (in the sense of laminar-turbulent transition at a Reynolds number of approximately 500). It is apparent from the foregoing that one who observes the flow only in one region is most likely to arrive at an isolated mechanism (e.g., instability, finite transition, impending stagnation, etc.) which might be only the subset of the over-all mechanism. However, our observations, only a small part of which could be reported herein, suggest that the over-all interpretation of the abrupt change of structure of the vortex core in a viscous flow is not ascribable to a single mechanism.

This investigation thus raises the possibility that the instability of flow and the finite transition may play significant roles in different regions of the Re-Ω domain and that the predominance of the effects of one mechanism in one region does not exclude the existence of a different mechanism in another region. In fact, these two mechanisms appear to be intermingling along the dividing region shown in Fig. 14. The change of the type of the breakdown form from spiral to axisymmetric (transition from region II to region I), in terms of a transition from an unstable state of flow to one associated with finite transition, appears to have been enunciated already by Hall.[13] According to him,

> "we could envisage a flow which develops with time in such a way that the core first becomes unstable in Ludwieg's sense, with spiral disturbances, and that the subsequent development of the unstable core, together with its interaction with the surrounding flow, leads to a state which is roughly axially symmetric and which may be associated with a finite transition."

As to the analysis of the instability which manifests itself in the form of a double or single spiral, one will have to consider the full equations of motion, together with various types of disturbances as cited in the introduction.

Conclusions

Two basic and conceptually different mechanisms govern the vortex breakdown phenomenon: hydrodynamic instability and finite-transition to a sequent state. Which mechanism will bring it about depends on the particular combination of the Reynolds and circulation numbers of the flow. Instability manifests itself more emphatically at low Reynolds and high circulation numbers. The finite-transition type of behavior is brought out more clearly in an unsteady swirling flow (such as the one created by the perturbation of circulation) than in a swirling steady flow. Additional conclusions regarding the nature of the primary flow, bubble shape and structure, and the two modes of spiral have been discussed in the text of the paper.

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Compressible Turbulent Boundary-Layer Heat Transfer to Rough Surfaces

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The semiempirical correlation of Owen and Thomson for heat transfer to rough surfaces in subsonic turbulent flow is extended to supersonic conditions. The correlation relates heat flux to skin friction, which is determined by Goddard's method for rough surfaces, modified empirically to include nonadiabatic wall conditions. The method is verified by comparison with existing experimental results for Mach numbers from 3 to 4.9. Application to a typical re-entry vehicle ablated nose shape indicates that the heat flux increases rapidly, then reaches a maximum, as roughness increases. The example indicates that Reynolds analogy is invalid for rough surfaces.

Nomenclature

B = sublayer Stanton number
C_f = local skin-friction coefficient
K = equivalent sand grain roughness height
M = Mach number
Pr = Prandtl number
R_N = nose radius
Re_K^* = roughness Reynolds number (Eq. 8)
Re_s = Reynolds number based on local properties and wetted length; $Re_s = \rho_e U_e S/\mu_e$
St = Stanton number
S = wetted length
T = temperature
U_τ = shear velocity; $U_\tau = (\tau_w/\rho_w)^{0.5}$
U_∞ = freestream velocity
α = empirical constant (Eq. 2)
θ = central angle; $\theta = S/R_N$
θ_N = nose semivertex angle
ρ = density
μ = viscosity

Subscripts

aw = adiabatic wall
e = edge of boundary layer
i = incompressible
O = smooth surface
R = recovery
W = wall

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Introduction

SURFACE roughness can cause significant increases in convective heat flux and skin friction. Recent developments in re-entry vehicle technology have indicated the importance of roughness in two regions: 1) ablative noses, and 2) ablative frustums having cross-hatching patterns. Materials such as graphite or ablative composites consisting of filler and binder materials can develop surface roughness of from 1 to 10 mils, which can be of the same order as the boundary-layer thickness on the nose during ballistic re-entry. This magnitude of roughness is sufficient to trip the laminar boundary layer to turbulent flow, as illustrated by the wind-tunnel data of Deveikis and Walker,[1] shown in Fig. 1. The increase in turbulent heat flux due to roughness is significant in determining the amount of ablation and shape change of the nose, as discussed by Welsh.[2] The cross-hatching ablation pattern phenomenon is common to the three major generic classes of heat shield materials (sublimers, melters, and charring ablators), and occurs in supersonic turbulent flow, as discussed by Laganelli and Nestler.[3] Much larger roughness amplitude of the wavy wall type can develop in cross-hatching ablation than in stagnation region